Finite Sample Properties of the Test for Long-Run Granger Non-Causality in Cointegrated Systems

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Abstract: It is known that the usual Wald type test is not applicable for the test of Granger non-causality in the long-run in cointegrated systems. The difficulty comes from the singularity of the relevant variance-covariance matrix. In order to circumvent the problem, we have recently proposed an alternative test statistic which can be approximated by a suitable chi-square distribution with a fractional degrees of freedom. The proposed test is very easy to implement in practical problems. In this paper, we present Monte Carlo simulations that reveal various aspects of the finite sample properties of the proposed method. In an empirical application, it is shown that real money does not cause the rest of variables in the long-run in a five variable system of Japanese macro-economy.

Keywords **:** Vector autoregression; Cointegration; Long-run causality; Hypothesis testing

1. INTRODUCTION

The Granger causality or non-causality has been one of main issues in time series analysis of economic data for past three decades. Tests for the Granger non-causality are straightforward in a stationary framework. In cointegrated systems, such tests are more complex, since the existence of unit roots gives various complications in statistical inference. See, for example, Toda and Phillips [1994]. In cointegrated systems we must distinguish the long-run Granger non-causality from the short-run one, while in stationary systems, we only have to be concerned with the short-run Granger non-causality. The definition of the long-run non-causality is given, for example, in Bruneau and Jondeau [1999]. Inference on the long-run causality has been known to be problematic when block non-causality is in question, since the variance-covariance matrix of the relevant coefficient matrix is generally degenerate.

Bruneau and Jondeau [1999] circumvented this degeneracy problem by focusing only on the non-causality from a variable to a variable rather than the block non-causality. Their method is difficult to generalised to the case of the long-run block non-causality. Yamamato and Kurozumi [2001] has proposed an alternative testing procedure for the long-run block non-causality. Their method circumvents the degeneracy problem by approximating its distribution with a suitable chi-square distribution. In this paper, we give the finite sample properties of the testing procedure by simulation experiments. Further, we apply the method to Japanase macroeconomic data, and find that real money does not Granger cause the rest of variables in the system in the long-run.

The remainder of the paper is organized as follows. In section 2 we briefly review the model and give the definitions of long-run Granger non-causality. Section 3 reviews the details of the testing procedure proposed by Yamamoto and Kurozumi [2001]. Section 4 gives the experimental results that reveal the finite sample properties of the proposed method. Section 5 gives a few empirical results on Japanese macro-variables. In section 6 we give a brief concluding remarks.

2. MODEL, ASSUMPTIONS, AND LONG-RUN NON-CAUSALITY

Consider m-vector process $\{x = [x_i]\}$ generated by vector autoregressive (VAR) model of order p,

$$A(L)x_t = \mu + \varepsilon_t \tag{1}$$

where $x_t = [x_{it}]$, $A(L) = I_m - A_1L - \cdots - A_pL^p$, L is the lag operator, I_m is the identity matrix of rank m, μ is the $m \times 1$ constant vector, $\{\varepsilon_t\}$ is a Gaussian white noise process with mean zero and nonsingular covariance matrix $\Sigma_{\varepsilon\varepsilon}$. Suppose that we know the true lag length p. Following Johansen [1995], we assume the following:

Assumption (Cointegration): *System* (1) *satisfies:*

(i) $|A(z)| = 0$ *has its all roots outside the unit circle or equal to* 1.

(ii) $\Pi = \alpha\beta'$, *where* $\Pi = -A(1)$, α *and* β *are* $m \times r$ *matrices of* rank r, $0 < r < m$, *and* $\text{rank}\{\Pi\} = r$. *Without loss of generality, it will be assumed that* β *is orthonormal.*

(iii) $\text{rank}\{\alpha'_\perp \Gamma \beta_\perp\} = m - r = s$, *where* $\alpha_\perp$ *and* $\beta_\perp$ *are* $m \times (m - r)$ *matrices such that* $\alpha'_\perp\alpha = 0$, $\beta'_\perp\beta = 0$, *and* $\Gamma = -(\partial A(z)/\partial z)_{z=1} - \Pi$.

These assumptions imply that each component of x_t is I(1), and linear combinations of $\beta' x_t$ are stationary. The components of x_t are cointegrated with the cointegrating matrix β and the cointegrating rank r. Subtracting x_{t-1} from both sides of (1) and rearranging the variables, we get the vector error correction (VEC) form of the process,

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \mu + \varepsilon_t \tag{2}$$

where $\Gamma_j = -\sum_{i=j+1}^{p} A_i \quad (j = 1, \cdots, p-1)$. The differenced process has representation

$$\Delta x_t = C(L)(\mu + \varepsilon_t) \tag{3}$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ with $C_0 = I_m$. Further, the vector moving average (VMA) representation of $\{x_t\}$ can be explicitly expressed as:

$$x_t = C\sum_{i=0}^{t} \varepsilon_i + C_1(L)\varepsilon_t + \tau t + x_0 - s_0 \tag{4}$$

where $C = [c_{ij}] = C(1) = \beta_\perp(\alpha'_\perp\Gamma\beta_\perp)^{-1}\alpha'_\perp$, $C_1(L) = (C(L) - C(1))/(1 - L)$, $\tau = C\mu$, $s_0 = C_1(L)\varepsilon_0$ such that $\beta' x_0 = \beta' s_0$. In the above representation (4), C is called the long-run impact matrix.

Next, we consider the companion form of the system (1) in order to express the long-run prediction explicitly.

$$X_t = \bar{A}X_{t-1} + \Xi_t \tag{5}$$

where $X'_t = [x'_t, x'_{t-1}, \cdots, x'_{t-p+1}]$, $\Xi'_t = [\varepsilon'_t, 0, \cdots, 0]$, $\bar{A} = \begin{bmatrix} A \\ \cdots\cdots\cdots \\ I_{(p-1)m} \;\vdots\; 0 \end{bmatrix}$, and $A = [A_1, A_2, \cdots, A_p]$, $A_1 = I_m + \alpha\beta' + \Gamma_1$, $A_i = \Gamma_i - \Gamma_{i-1}$ $(i = 2, \cdots, p-1)$, $A_p = -\Gamma_{p-1}$. The h-th period ahead least squares prediction of x_{t+h} given X_t is given by:

$$x_{t+h\,|\,t} = M'\bar{A}^h X_t \equiv B_h X_t$$

where $B_h = M'\bar{A}^h$, and $M' = [I_m, 0, \cdots, 0]$. The long-run prediction is defined as the one when the prediction horizon h goes to infinity. It is known that B_h converges to a non-zero finite matrix as h goes to infinity. [See, for example, Phillips, 1998]. The coefficient matrix of the long-run prediction is defined as:

$$\bar{B} = [\bar{b}_{ij}] = [\bar{B}_1, \bar{B}_2, \cdots, \bar{B}_p] = \lim_{h\to\infty} B_h \tag{6}$$

The hypothesis of the block long-run non-causality from $R^{*\prime}_R x$ to $R_L x$ is defined in terms of $\bar{B}$ as:

$$H_0 : R_L\bar{B}R_R = 0 \tag{7}$$

where $R_L = [0, I_{p_1}, 0]$, $R_R = I_p \otimes R^*_R$, and $R^*_R = [0, I_{p_2}, 0]'$ with either $p_1 \geq 2$ and/or $p_2 \geq 2$. For example, when the first variable does not cause the rest of variables in the system, the choice matrices are given by $R_L = [0, I_{m-1}]$ and $R^*_R = [1, 0, \cdots, 0]'$. The above definition of the block long-run non-causality is a straightforward generalization of Bruneau and Jondeau [1999] who discuss the case where $p_1 = p_2 = 1$.

It is easily seen that we have $\bar{B}_1 = C$, where C is the long-run impact matrix defined in (4). [See, for example, Phillips, 1998]. The closely related concept to the long-run non-causality is the long-run neutrality. While there are various definition of long-run neutrality in the literature, according to Bruneau and Jondeau [1999], the hypothesis of the long-run neutrality of $R^{*\prime}_R x$ to $R_L x$ is given by:

$$H_1 : R_L\bar{B}R_R = R_L\bar{B}_1R^*_R = R_LCR^*_R = 0 \tag{8}$$

where $R_R = e_p \otimes R^*_R$, and e_p is the $p \times 1$ vector such that $e_p = [1, 0, \cdots, 0]'$. Needless to say, the long-run neutrality is a necessary, but not a sufficient, condition for the long-run causality.

3. TEST FOR LONG-RUN NON-CAUSALITY VIA APPROXIMATE CHI-SQUARE DISTRIBUTION

In this subsection, we review the new testing procedure by Yamamoto and Kurozumi [2001]. In order to test the hypothesis (7), we first estimate the VEC form (2) of the process by the ML method. The coefficients of the levels VAR form (1) are constructed from the VEC estimates. The asymptotic distributions of coefficient matrices of the h-period ahead prediction $\hat{B}_h$ and the long-run prediction $\hat{\bar{B}}$ are given in the following Proposition.

Proposition 1: Let Assumption holds and let $\hat{B}_h$ be estimates of the least squares prediction matrix B_h obtained from the ML estimates on the VEC representation (2).

(i) *For fixed h, we have*

(a) $\hat{B}_h \xrightarrow{p} B_h$, *and*

(b) $\sqrt{T}\,vec(\hat{B}_h - B_h) \xrightarrow{d} N(0, \Sigma_h)$,

where vec(·) *is the row-stacking operator,* $\Sigma_h = F_h \Sigma_{vec} F_h'$, $\Sigma_{vec} = \Sigma_{\varepsilon\varepsilon} \otimes \Sigma_{\xi\xi}^{-1}$, $\Sigma_{\xi\xi} = E[\xi_t \xi_t']$, $\xi_t = [(\beta' x_{t-1})', \Delta x_{t-1}', \cdots, \Delta x_{t-p+1}']'$, $F_h = \sum_{i=0}^{h-1} C_i \otimes \bar{A}'^{h-1-i} K'^{-1} G_\xi$, $C_i = M' \bar{A}^i M$ *is the i-th impulse response matrix,*

$$G_\xi = \begin{bmatrix} \beta & 0 \\ 0 & I_{(p-1)m} \end{bmatrix} \quad \text{and}$$

$$K^{-1} = \begin{bmatrix} I_m & & & & 0 \\ I_m & -I_m & & & \\ & \cdot & \cdot & & \\ & & \cdot & \cdot & \\ 0 & & & \cdot & \cdot \\ & & & I_m & -I_m \end{bmatrix}$$

(ii) *If $h \to \infty$ as $T \to \infty$ with either $h = fT$ or $h/T \to 0$ where $f > 0$ is a fixed fraction of the sample, we have*

(a) $\hat{\bar{B}}_h \xrightarrow{p} \bar{B}$ *and*

(b) $\sqrt{T}\,vec(\hat{\bar{B}} - \bar{B}) \xrightarrow{d} N(0, \Sigma)$

where $\Sigma = F\Sigma_{vec}F'$, $F = C \otimes K'^{-1} GL \times (I_{(p-1)m+r} - E_{22}')^{-1} \begin{bmatrix} I_r & 0 \\ 0 & I_{p-1} \otimes H' \end{bmatrix}$, $G = I_p \otimes H$, $H = [\beta_\perp, \beta]$, $L' = [0, I_{(p-1)m+r}]$ *and*

$$E_{22} = \begin{bmatrix} I_r + \beta'\alpha & \dddot{\Gamma}_1 & & \dddot{\Gamma}_{p-1} \\ \beta_\perp'\alpha & \ddot{\Gamma}_1 & & \ddot{\Gamma}_{p-1} \\ \beta'\alpha & & & \\ I_m & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \cdot & & \ddots & \\ 0 & 0 & I_m & 0 \end{bmatrix}$$

and $\bar{\Gamma}_i = H'\Gamma_i H = [\ddot{\Gamma}_i', \dddot{\Gamma}_i']'$, $\ddot{\Gamma}_i = \beta_\perp' \Gamma_i H$, *and* $\dddot{\Gamma}_i = \beta' \Gamma_i H$ $(i = 1, 2, \cdots, p-1)$.

Proof: See Yamamoto and Kurozumi [2001]. See also Phillips [1998] and Arai and Yamamoto [2000].

Then, we have, under H_0,

$$\sqrt{T}\, R\, vec\{\hat{\bar{B}} - \bar{B}\} \xrightarrow{d} N(0, R\Sigma R')$$

where $R = R_L \otimes R_R'$ is the $p_1 p_2 \times m^2 p$ matrix. It should be noted that the usual Wald type test statistic, under H_0,

$$W = T\{R\, vec(\hat{\bar{B}})\}'(R\Sigma R')^{-1}\{R\, vec(\hat{\bar{B}})\} \quad (9)$$

is generally infeasible, because $R\Sigma R'$ is not necessarily non-singular. The degeneracy of $R\Sigma R$ comes from that of Σ.

$$\Sigma = F\Sigma_{vec}F' = C\Sigma_{\varepsilon\varepsilon}C' \otimes P\Sigma_{\xi\xi}^{-1}P' \quad (10)$$

We may note that both $C\Sigma_{\varepsilon\varepsilon}C'$ and $P\Sigma_{\xi\xi}^{-1}P'$ are degenerate, because $C = \beta_\perp(\alpha_\perp' \Gamma \beta_\perp)^{-1}\alpha_\perp'$ is obviously degenerate by its construction, and P is the $mp \times \{(p-1)m + r\}$ matrix.

In order to circumvent the problem, Yamamoto and Kurozumi [2001] has proposed an alternative test statistic, under H_0,

$$W^+ = T(R\hat{b})'(R\hat{b}) \quad (11)$$

where $\hat{b} = vec(\hat{\bar{B}})$. In general, the exact distribution of W^+ depends on the nuisance parameters and it is tedious to derive it for practitioners. Instead, they approximate the distribution of W^+ by $a\chi_f^2$, as discussed in Chapter 29 of Johnson and Kotz [1970] and Satterthwaite [1941], where χ_f^2 is a chi-square distribution with f degrees of freedom, with a and f chosen to make the first two moments in agreement with those of W^+. Thus, we have, under H_0,

$$W^+ = T(R\hat{b})'R\hat{b} \underset{\text{approx.}}{\sim} a\chi_f^2 \quad (12)$$

where $a = \sum_{j=1}^{s} \lambda_j^2 / (\sum_{j=1}^{s} \lambda_j)$, $s = p_1 p_2$ and $f = (\sum_{j=1}^{s} \lambda_j)^2 / (\sum_{j=1}^{s} \lambda_j^2)$, and λ_j $(j = 1, 2, \cdots, s)$ are the characteristic roots of $R\Sigma R'$. Note that, in general, the degrees of freedom, f, is fractional and significance points for χ^2 with degrees of freedom differing by 0.2 are given in Pearson and Hartley [1976]. Further, the computer package GAUSS has a convenient built-in function called "cdfchinc" which returns a p-value for a chi-square distribution with a fractional degrees of freedom. We will use it in the experiments and applications later.

4. FINITE SAMPLE EXPERIMENTS

In this section, we examine the finite sample performance of the test reviewed in the previous section.

4.1 The Case of $p = 1$

In this sub-section, we consider the following simple VEC form of the model with $m = 3$ and $p = 1$:

$$\Delta x_t = \alpha\beta' x_{t-1} + \varepsilon_t \,, \qquad (13)$$

where $\{\varepsilon_t\}$ is i.i.d. $N(0, I_3)$. In this case, the test for long-run non-causality is the same as the test for long-run neutrality. The nominal significance level is 5% and the number of replication is 1000 throughout the experiments.

The first model with $r = 2$ is an example of long-run neutrality of x_1, under H_0, and is described as:

$$\alpha = [\alpha_{ij}] = \begin{bmatrix} -1 & 0.2 \\ 0 & -0.5 \\ 0 & -1.0 \end{bmatrix} \quad \text{and} \qquad (14)$$
$$\beta = [\beta_{ij}] = \begin{bmatrix} 0.5 & -0.5 \\ -0.4 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

This gives the following long-run impact matrix:

$$C = [C_{ij}] = \begin{bmatrix} 0.0 & 0.719 & -0.360 \\ 0.0 & 0.719 & -0.360 \\ 0.0 & -0.144 & 0.072 \end{bmatrix} \qquad (15)$$

It represents the case where x_1 does not cause the system in the long-run. We test the hypothesis H_0 with W^+ in (11) where $R_L = I_3$, and $R_R = R_R^* = [1, 0, 0]'$. Table 1 shows the empirical size and power of the test for $T = 100$, 200, 400, and 1000, where T is the sample size. When α_{21} of α is 0, it shows the empirical size. It appears that the empirical size is slightly greater than the nominal size, when $T = 100$. But its size distortion diminishes smoothly as T increases. It also shows the empirical power of the test when α_{21} of α is set to 0.1, 0.2 , and 0.3 . Accordingly, (c_{11}, c_{21}, c_{31}) are changed to (0.064, 0.064, −0.014), (0.155, 0.155, −0.031), and (0.216, 0.216, −0.043), respectively. The results indicate that the empirical power smoothly increases as the first column of C deviates away from the null vector.

Table 1. Empirical Size and Power: First Model.

$\alpha_{21} \setminus T$	Rejection Percentages 100	200	400	1000
	Size			
0	6.8	5.7	5.0	5.4
	Power			
0.1	20.3	27.3	42.3	76.1
0.2	45.3	67.8	91.3	100.0
0.3	72.0	92.2	97.7	100.0

The second model with $r = 1$ is drawn from Paruolo [1997], which is originally the $N1$ model in Toda and Phillips [1994]:

$$\alpha = \begin{bmatrix} -0.5 \\ -0.5 \\ 0 \end{bmatrix} \quad \text{and} \quad \beta = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \qquad (16)$$

This gives the following long-run impact matrix, under under H_0,

$$C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad (17)$$

It represents a typical block non-causality from x_1 and x_2 to x_2 and x_3 in the long-run. We test the hypothesis H_0 with W^+ in (11) where $R_L = [0, I_2]$, and $R_R' = R_R^{*\prime} = [I_2, 0]$. Table 2 shows the empirical size and the power of the test for $T = 100, 200, 400$ and 1000. The performance of the size of the test, when $\alpha_3 = \beta_1 = 0$ is fairly good. The second and third parts of table 2 gives the empirical power of the test when α or β is changed and accordingly C matrix is changed. First, α_3 in $\alpha = [-0.5, -0.5, \alpha_3]'$ is set to 0.05, and 0.1. Accordingly, (c_{22}, c_{32}) are changed to (0.167, 0.083), and (0.286, 0.143), while (c_{21}, c_{31}) stay unchanged. Second, β_1 in $\beta' = [\beta_1, 1, -2]$ is set to 0.05, 0.1, and 0.15. Accordingly, (c_{21}, c_{22}) are changed to (−0.048, 0.048), (−0.091, 0.091), and (−0.130, 0.130), while (c_{31}, c_{32}) stay unchanged. The results in Table 2 shows that the empirical power is more sensitive to changes in α_3 than in β_1.

Table 2. Empirical Size and Power: Second Model.

α_3	$\beta_1 \setminus T$	Rejection Percentages 100	200	400	1000
		Size			
0	0	7.2	6.1	5.6	4.6
		Power			
0.05	0	39.0	54.5	78.3	100.0
0.1	0	81.1	95.7	99.9	100.0
		Power			
0	0.05	11.7	11.8	13.7	25.9
0	0.10	17.7	23.4	40.8	96.1
0	0.15	28.7	46.3	87.6	100.0

4.2 The Case of $p = 2$

In this sub-section, we consider the third model which is given as a VEC model with $m = 3$ and $p = 2$:

$$\Delta x_t = \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \varepsilon_t \tag{18}$$

where $\{\varepsilon_t\}$ is i.i.d. $N(0, I_3)$.

Under H_0, α and β are set to be the same as (14) in the first model. Further, Γ_1 is specified as

$$\Gamma_1 = \begin{bmatrix} 0.0 & -0.3 & -0.1 \\ \gamma_{21} & -0.4 & -0.2 \\ \gamma_{31} & -0.3 & -0.2 \end{bmatrix} \tag{19}$$

Under H_0, we assume that $\gamma_{21} = \gamma_{31} = 0$. It represents the case where x_1 does not cause x_2 and x_3 in the long-run.

We test the hypothesis H_0 with W^+ in (11) where $R_L = [0, I_2]$, and $R_R^* = [1, 0, 0]'$. As a by-product, we can also test the hypothesis that x_1 is neutral to x_2 and x_3 in the long-run. Table 3 shows the empirical size and power of the test for $T = 100$, 200, 400, and 1000. It appears that the empirical size is slightly greater than the nominal size, when $T = 100$ or 200. But its size distortion diminishes as T increases to 1000. The second part of Table 3 gives the empirical power of the test when α_{21} of α is set to $0.1, 0.2$, and 0.3. Accordingly, (c_{21}, c_{31}) of the long-run impact matrix are changed to (0.068, −0.014), (0.125, −0.025), and (0.173, −0.035), respectively. It shows that the empirical power increases smoothly as c_{21} and/or c_{31} deviate from the null vector. It shows that there is not much difference in size and power between the long-run non-causality test and the long-run neutrality test, although the power is slightly higher for the test of the neutrality. The third part of Table 3 also gives the empirical power of the test when the parameters of stationary part of the model are changed. Namely, γ_{21} and γ_{31} of Γ_1 in (19) are set to $-0.1, -0.2$, and -0.3. Accordingly, $(\bar{b}_{24}, \bar{b}_{34})$ of the long-run prediction matrix $\bar{B}$ are changed to (0.036, −0.007), (0.069, −0.014), and (0.101, −0.020), respectively. It shows that the empirical power is not strong when the stationary part of the model changes, unless the sample is 400 or 1000. Note that in this case the long-run impact matrix C does not change. Thus, the column of the rejection percentage of neutrality gives the empirical size rather than the empirical power.

In conclusion, these experiments suggest that the proposed test for long-run Granger non-causality shows a good performance in the empirical size and power. In term of the empirical power, the changes in the stationary part, give weaker effects compared with the changes in the long-run parameters.

Table 3. Empirical Size and Power: Third Model.

α_{21}	$\gamma_{21} = \gamma_{31}$	T	Rejection Percentages Causality	Neutrality
			Size	
0	0	100	5.9	6.5
		200	6.1	6.6
		400	5.6	5.5
		1000	4.7	4.7
			Power	
0.1	0	100	15.1	15.9
		200	19.9	20.1
		400	25.9	26.5
		1000	50.5	51.9
0.2	0	100	28.5	31.4
		200	44.1	44.9
		400	66.8	68.3
		1000	95.1	95.8
0.3	0	100	45.4	47.2
		200	70.9	71.4
		400	91.7	91.8
		1000	100.0	100.0
			Power	Size
0	−0.1	100	6.8	6.7
		200	7.4	6.6
		400	6.8	5.6
		1000	10.0	4.8
0	−0.2	100	8.8	6.8
		200	10.7	6.5
		400	16.6	5.4
		1000	53.7	4.9
0	−0.3	100	12.1	6.7
		200	19.6	6.1
		400	46.5	5.5
		1000	98.5	4.9

5. EMPIRICAL APPLICATIONS

In this section, we examine the long-run non-causality between money and real variables in Japan. We consider five variable VEC estimation which involves, in order of appearance in the VAR model, real $M2 + CD$, m_t, real income y_t, real wealth other than $M2 + CD$, w_t, the own interest rate, Rm_t, and the rival interest rate, Rr_t. The sample period is from 1975(1) to 1994(4). In the preliminary ADF unit root test, all but Rm_t and Rr_t are not rejected for a unit root hypothesis. In the VAR estimation, a constant term is included and the lag length of 4 is selected by the AIC criterion. Results of the trace test and the maximum eigenvalue test indicate that the cointegration rank is 3 at 10 percent significance level, and is 1 at 5% significance level. We discard the case of only one cointegration, since it is inconsistent with the fact that Rm_t and Rr_t are not integrated. The estimates of the adjustment coefficients α, the cointegration vectors β, and the estimate of the long-run impact matrix C are omitted in lieu of space.

We first test the hypothesis that real money does not Granger cause the rest of variables in the system in the long-run. In this case, the restriction matrices are given as $R_L = [0, I_4]$, and $R_R = [1, 0, 0, 0, 0]'$. We get the test statistic $W^+ = 1.68$, which gives a p-value of 0.582. Thus, we cannot reject H_0. It means that a change in real money m_t will not affect any variable in the system in the long-run.

Next, we test the long-run non-causality of the wealth w_t toward real money m_t and income y_t. The restriction matrices are given as $R_L = [I_2, 0]$, and $R_R = [0, 0, 1, 0, 0]'$. We get $W^+ = 6.35$, which gives a p-value of 0.0213. Thus, we reject H_0 at 5 percent significance level and we conclude that w_t does Granger cause m_t and p_t in the long-run.

The present samle size is admittedly small, and the investigation with the extended sample is under way.

6. CONCLUSION

In this paper, we have conducted the small sample experiments for the newly proposed test for the Granger non-causality in the long-run in cointegrated systems. The test circumvents the problem of degeneracy of the variance-covariance matrix associated with the usual Wald type test by approximating it with a suitable chi-square distribution. The experiments indicate that the proposed method perform reasonably well in finite samples. As an empirical application, we find that real money does not cause the rest of variables in a five variable system of Japanese macro-economy.

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